CAPACITANCE

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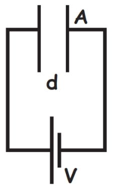


$Q=CV$

$C=\frac{Q}{V}$

$C=\frac{\varepsilon_0 A}{d}$

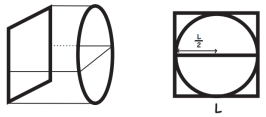
Capacitance Depends on

1. Distance betweent plates
2. Area of plates
3. Medium b/w plates

Capacitance is Independent of

1. Charge
2. Potential difference

Unit Of Capacitance = Farad



$A_{eff}=\pi\left(\frac{L}{2}\right)^2=\pi\frac{L^2}{4}$

$C=\frac{\varepsilon_0 A_{eff}}{d}=\frac{\pi\varepsilon_0 L^2}{4d}$

ENERGY STORED

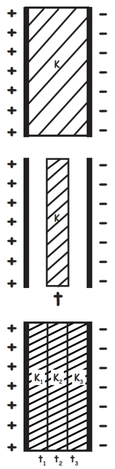
Work done by battery $=QV$ (100%)

- 50% Energy stored in capacitor $=\frac{1}{2}QV=\frac{CV^2}{2}=\frac{Q^2}{2C}$
- 50% Energy dissipated $=\frac{1}{2}QV=\frac{CV^2}{2}=\frac{Q^2}{2C}$

Variation with plate seperation

1. $Q=CV=\frac{\varepsilon_0 AV}{d}$
2. $C=\frac{\varepsilon_0 A}{d}$ $\quad C\propto\frac{1}{d}$
3. $V=\frac{Q}{C}=\frac{Qd}{\varepsilon_0 A}$ $\quad$ 4. $E=\frac{Q}{A\varepsilon_0}=\frac{V}{d}$
5. $F=\frac{Q^2}{2A\varepsilon_0}=\frac{CV^2}{2d}$
6. $U=\frac{CV^2}{2}$ OR $U=\frac{Q^2}{2C}=\frac{Q^2 d}{2\varepsilon_0 A}$ $=\frac{\varepsilon_0 AV^2}{2d}$

DIELECTRIC IN CAPACITOR



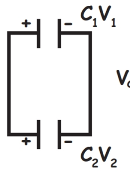
$C'=\frac{k\varepsilon_0 A}{d}$

$C'=KC$

$C'=\frac{\varepsilon_0 A}{d-t+\frac{t}{K}}$

$C'=\frac{\varepsilon_0 A}{\frac{t_1}{K_1}+\frac{t_2}{K_2}+\frac{t_3}{K_3}}$

Connecting two charged capacitors - Case 1



$V_{common}=\frac{C_1V_1+C_2V_2}{C_1+C_2}$

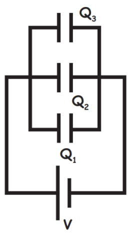
Initial Energy → Heat loss → **Final Energy**

$U_i=\frac{C_1V_1^2}{2}+\frac{C_2V_2^2}{2}$

$U_f=\frac{1}{2}(C_1+C_2)\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right)^2$

Energy loss $=\frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$

2. Parallel combination



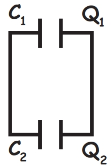
$C_{eq}=C_1+C_2+C_3$

$C_{eq}=nC$

[if $C_1=C_2=.......=C_n=C$]

Charge divider rule

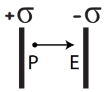
$Q\propto C$



$Q_1=\left(\frac{C_1}{C_1+C_2}\right)Q$

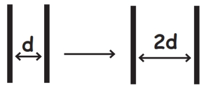
$Q_2=\left(\frac{C_2}{C_1+C_2}\right)Q$

ELECTRIC FIELD



$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{A\varepsilon_0}=\frac{CV}{A\varepsilon_0}$

when plate seperation doubled

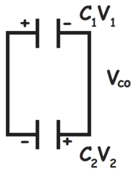


$C=\frac{\varepsilon_0 A}{d}$ $\quad C'=\frac{C}{2}$

Dielectric inserted in capacitor

Battery removed	Battery remains connected
1. $C=KC'$	1. $C'=KC$
2. $Q'=Q$	2. $Q'=KQ$
3. $V'=\frac{V}{K}$	3. $V'=V$
4. $E'=\frac{E}{K}$	4. $E'=E$
5. $U'=\frac{U}{K}$	5. $U'=KU$

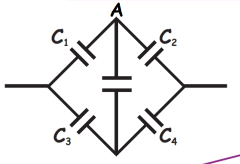
Connecting two charged capacitors - Case 2



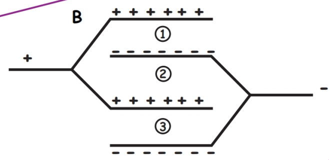
$V_{common}=\frac{C_1V_1-C_2V_2}{C_1+C_2}$

Energy loss $=\frac{C_1C_2}{2(C_1+C_2)}(V_1+V_2)^2$

WHEATSTONE'S BRIDGE



$\frac{C_1}{C_2}=\frac{C_3}{C_4}$ $\quad V_A=V_B$

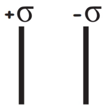


$C=\frac{3\varepsilon_0 A}{d}$

POTENTIAL

$V=Ed=\frac{Qd}{A\varepsilon_0}$

Force b/w plates of a parallel plate capacitor



$F=Q\times E_{\text{Due to one plate}}$

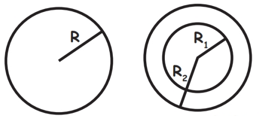
$=Q\times\frac{\sigma}{2\varepsilon_0}=Q\times\frac{Q}{2A\varepsilon_0}$

$=\frac{Q^2}{2A\varepsilon_0}=\frac{C^2V^2}{2A\varepsilon_0}=\frac{\varepsilon_0 A}{d}\times\frac{CV^2}{2A\varepsilon_0}$

$=\frac{CV^2}{2d}$

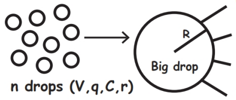
Battery connected	Battery Disconnected
$d\to 2d$	$d\to 2d$
1. $V=$Constant	1. $Q=$Constant
2. $C'=\frac{C}{2}$	2. $C'=\frac{C}{2}$
3. $Q=CV$, $Q'=\frac{Q}{2}$	3. $V=\frac{Q}{C}$, $V'=2V$
4. $E=\frac{V}{d}$, $E'=\frac{E}{2}$	4. $E=\frac{Q}{A\varepsilon_0}=$ Constant
5. $F=\frac{CV^2}{2d}$, $F'=\frac{F}{4}$	5. $F=\frac{Q^2}{2A\varepsilon_0}=$ Constant
6. $U=\frac{CV^2}{2}$, $U'=\frac{U}{2}$	6. $U=\frac{Q^2}{2C}$, $U'=2U$

SPHERICAL CAPACITOR



$C=4\pi\varepsilon_0 R$ $\quad C=4\pi\varepsilon_0\left[\frac{R_1R_2}{R_2-R_1}\right]$

REDISTRIBUTION OF CHARGE



$Q'=nq$

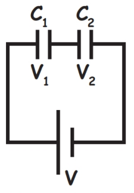
$C'=n^{1/3}C$

$V'=n^{2/3}V$

$U'=n^{5/3}U$

Grouping of capacitors

1. Series combination



$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}$

$V=V_1+V_2$

$C_{eq}=\left(\frac{C_1C_2}{C_1+C_2}\right)$

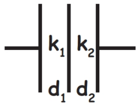
Voltage divider rule

$V\propto\frac{1}{C}$

$V_1=\left(\frac{C_2}{C_1+C_2}\right)V$ $\quad V_2=\left(\frac{C_1}{C_1+C_2}\right)V$

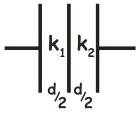
MULTIPLE DIELECTRICS

1. Series combination



$k_{eq}=\frac{\Sigma d}{\Sigma d/k}$

$k_{eq}=\frac{d_1+d_2}{\frac{d_1}{k_1}+\frac{d_2}{k_2}}$

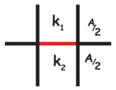


$k_{eq}=\frac{2k_1k_2}{k_1+k_2}$

2. Parallel combination

$k_{eq}=\frac{\Sigma kA}{\Sigma A}$

$k_{eq}=\frac{k_1A_1+k_2A_2+....}{A_1+A_2+....}$



$k_{eq}=\frac{k_1\frac{A}{2}+k_2\frac{A}{2}}{\frac{A}{2}+\frac{A}{2}}=\frac{k_1+k_2}{2}$